

CTR-Averse Betting by James Grosjean and Previn Mankodi

The Irrelevance of the “Trip-Bankroll Problem”

Players frequently ask “How much of my bankroll should I bring on a trip?” Or, “If I bring \$X, what is my trip-ruin probability?” Along these lines, some analysis has been done on the question, “If I have only \$X with me, out of a much larger bankroll, how should I bet?” Except possibly for a team with a bankroll so large that traveling with it would be logistically difficult (or expensive due to the opportunity cost of keeping the bankroll in cash), we consider these questions to be misguided, perhaps irrelevant. Our recommendation is that a player on a trip should have access to enough funds so that his trip-ruin probability is zero! To bust out on one trip in a career is too many. As a practical matter, additional funds are often only a phone call away if other pros are in the area. For those seeking rules of thumb, here’s one: a player with a bankroll of \$30000 or less should bring it all.

If these recommendations are followed, a player might never encounter a trip-bankroll problem. Sadly, all of our handwaving cannot make the problem disappear, because it rears its ugly head in a mathematically identical manifestation: Regulation 6A. Legally, Reg. 6A is Nevada’s version of what most of us refer to as the “CTR Rule,” which requires a person to provide identification for a Currency Transaction Report if he makes cash transactions exceeding \$10000 in a 24-hour period. The casino will demand ID in order to complete this form for the U.S. Treasury Department (not the IRS). Failure to provide ID will result in a “CTR backoff,” meaning that the casino is required to refuse the player’s action until ID is provided.

So, a player who is unwilling to provide ID will have to quit a game before exceeding a \$10000 loss. This is mathematically equivalent to having a \$10000 trip bankroll. While it is easy to alleviate the trip-bankroll problem by bringing more money, this \$10000 barrier is a bit more stubborn. Our task is to determine how this constraint affects our betting strategy.

Presumably, the player’s full bankroll is large, so this is not an issue of risk aversion where the player must avoid busting out at all costs (a la Kelly). This is an issue of expectation maximization. Our player is not afraid to lose the \$10000, except insofar as busting out ends his game prematurely, and might produce a smaller expected profit than a betting strategy where he bets conservatively to avoid busting out. Let’s look at a contrived example to see the phenomenon.

Scaling Back because of a Table Limit

Suppose the player may bet on a fair coin paying 2:1 on winning wagers. The expectation of this game is $0.5(2) + 0.5(-1) = 0.5$, with variance $0.5(4) + 0.5(1) - 0.5^2 = 2.25$. He has a bankroll of \$1 million at home, but only \$5 cash in his pocket. His Kelly wager would be \$222,222.22, representing a fraction $E/V = 0.5/2.25 = 22.\bar{2}\%$ of his bankroll. If he is allowed to bet on three coin flips, he would bet his \$5, and then continue to fully parlay any winnings.

His betting decisions, expected action, and expected profit are shown in the first section of the following table. His expected action is \$23.75. He busts out a lot, but so what? A player who is afraid of busting out, betting only \$3 on the first round, will not do better. This player puts less money into action on the round he holds back, and since the game is positive, the guy who bets the max will have, in expectation, even more funds available to put into action on the subsequent round. Holding back accomplishes nothing other than lowering the expected profit. We’re here to make money, not worry about busting out. (For those who don’t want to bust out: don’t play!) When we say “bust out,” we are referring only to the player’s cash on hand; his full bankroll is huge, and not at risk. A full Kelly wager would be far in excess of anything he will wager here.

Betting Strategy in Three-Flip Game 50% Edge, \$5 Cash on Hand									
Flip Out-comes	No Limits		With \$5 Table Maximum			With \$10 Win Ceiling			
	Bet Max	Hold Back	Bet Max	Hold Back	Bet Best	Bet Max	Hold Back	Bet Best	Bet Best
LLL	5/-/- -5	3/2/- -5	5/-/- -5	3/2/- -5	3.3̄/1.6̄/- -5	5/-/- -5	3/2/- -5	3.3̄/1.6̄/- -5	2.5/2.5/- -5
LLW	5/-/- -5	3/2/- -5	5/-/- -5	3/2/- -5	3.3̄/1.6̄/- -5	5/-/- -5	3/2/- -5	3.3̄/1.6̄/- -5	2.5/2.5/- -5
LWL	5/-/- -5	3/2/6 -5	5/-/- -5	3/2/5 -4	3.3̄/1.6̄/5 -5	5/-/- -5	3/2/4.5 -3.5	3.3̄/1.6̄/5 -5	2.5/2.5/3.75 -1.25
LWW	5/-/- -5	3/2/6 13	5/-/- -5	3/2/5 11	3.3̄/1.6̄/5 10	5/-/- -5	3/2/4.5 10	3.3̄/1.6̄/5 10	2.5/2.5/3.75 10
WLL	5/15/- -5	3/11/- -5	5/5/5 0	3/5/5 -4	3.3̄/5/5 -3.3̄	5/-/- 10	3/2/3 1	3.3̄/1.6̄/2.5 2.5	2.5/2.5/3.75 -1.25
WLW	5/15/- -5	3/11/- -5	5/5/5 15	3/5/5 11	3.3̄/5/5 11.6̄	5/-/- 10	3/2/3 10	3.3̄/1.6̄/2.5 10	2.5/2.5/3.75 10
WWL	5/15/45 -5	3/11/33 -5	5/5/5 15	3/5/5 11	3.3̄/5/5 11.6̄	5/-/- 10	3/2/- 10	3.3̄/1.6̄/- 10	2.5/2.5/- 10
WWW	5/15/45 130	3/11/33 94	5/5/5 30	3/5/5 26	3.3̄/5/5 26.6̄	5/-/- 10	3/2/- 10	3.3̄/1.6̄/- 10	2.5/2.5/- 10
Expected Action	190/8	154/8	80/8	82/8	83.3̄/8	40/8	55/8	55/8	55/8
Expected Profit	95/8	77/8	40/8	41/8	41.6̄/8	20/8	27.5/8	27.5/8	27.5/8
Table shows the bets, separated by slashes, for each coin flip. The profit for each case is shown below the bets. The bottom two lines of the chart show the expected action and expected profit derived from each betting strategy. Since each of the eight flip combinations is equally likely, the values in the bottom rows are shown in eighths.									

Now suppose we impose a table maximum bet of \$5. The expectation-maximizing guy was going to bet \$5 on the first round (well below his Kelly wager, but remember he has only \$5 in his pocket). The table maximum does not restrict his desired first-round bet, so should he not go ahead and bet his \$5 as before? If he does so and loses, he is finished, just like before. If he wins the first flip, he then has enough to bankroll the remaining two flips to the table maximum of \$5. His expected profit is \$5 on expected action of \$10.

Look again at the guy who holds back \$2 on the first flip and bets \$3. Subsequently, he bets to the limit—either his remaining bankroll or the table maximum. His expected action is \$10.25; he has surpassed the max bettor! The motivation of the max bettor is that by betting everything now in a positive game, he has even more funds available to bet on the subsequent hand. He then goes all in on the next hand, and so on; no other player can ever catch up to his action. When the table maximum is imposed, there is less benefit to betting the max now. The guy who bets only \$3 will have enough, if he wins the first flip, to max out the remaining two hands. Betting the max will not allow a player to get any more into action than that. With no limits, a dollar won on the first round had “value” greater than a dollar, because it could be parlayed in this positive-expectation game. That is no longer true; it is as if we have reduced the payoff on a winning bet. The effect is similar to games that have a maximum aggregate payoff, such as Let It Ride.

The guy who holds back two dollars gives up action, equivalently profit, in the case where the first flip is won. When the first flip is lost, the guy who has \$2 left then catches up in the cases where the second flip wins, keeping him in action until the third flip, while the max bettor is already long gone.

The two betting strategies are compared in the middle section of the table. Also shown is the optimal betting strategy, which recommends that the player bet $3.3\bar{3}$ on the first flip. This is a clever bet, and one that may appear familiar to tournament players. If he wins, he has enough to bet the table maximum on the subsequent two flips. If he loses, he can then bet $1.6\bar{6}$, which if won would leave him with enough funds for one max bet on the last hand.

The bottom line is that holding back a bit leads to a higher expected profit. Remember that

this is not a question of risk aversion or fear of busting out. We don't care about busting out per se, but it turns out that betting conservatively allows the player in this example to get more money into action. This principle even trumps Kelly! This table maximum is well below the guy's Kelly wager, and he has no fear of busting out the session, but he nevertheless bets well below the table maximum on the first hand. Is it not curious that the imposition of a \$5 table maximum on a player who was going to bet \$5 now causes him to bet \$3.33?

In most cases, the table maximum does not cause the player to hold back much. He will often just bet the table maximum. There is, however, a more powerful constraint that will cause significant reductions in bet sizing.

Scaling Back because of a Win Ceiling

In most casino contexts, there is a ceiling on the amount that can be won in a session, due to heat. In some sweaty casinos, a player would be prudent to quit after a win of \$4000 or \$5000. In an action joint, perhaps a player could win \$100000 in a session, or even \$225000. The amount that can be won depends on the casino size, the pit personnel, the skill of the BP, the BP's account history, the skill of the spotter, the game being played, the amount of concurrent civilian action, and other factors. Weighing these factors, a crew usually has an idea, going into a game, of how much can be won, and when they will quit, even in the absence of visible heat. Another interpretation is that a player will be backed off after a certain amount is won.

In the analysis that follows, we impose a "hard cap" on how much can be won. A hard cap means that a player may not wager an amount if winning that bet would put his aggregate win over the ceiling. A soft cap, in contrast, would allow the player's current hand to propel him beyond the ceiling, at which point he would then quit before playing any additional hands. A soft cap may induce betting schemes that have a player creep up close to the ceiling, and then make monster bets that propel him as far past the ceiling as possible if he wins, or back to the starting point if he loses. The difference between a hard cap and a soft cap is mitigated in our analysis by the simultaneous presence of a table maximum on each hand.

Let's look at the three-flip example, but impose a win ceiling of \$10. There is no table maximum. The player who bets \$5 on the first flip will again be finished after one flip, with \$5 of action. A player who bets \$3 on the first flip can do much better, generating \$6.875 in action (55/8). It turns out that this betting scheme, shown in the table is optimal, though not unique. Two other optimal strategies are shown. Note how the bets are chosen so that the player can exactly touch the ceiling. The optimal strategies all call for an initial bet in the range \$1.66 through \$3.33.

The win ceiling is really a conditional table maximum. It merely requires that depending on previous wins and losses, the current bet might be restricted. Under this interpretation, once the player has reached the win ceiling, it is as if we then impose a table maximum for all subsequent rounds of \$0—he cannot bet anymore. In general, the win ceiling is what drives the reduction in bets.

Scaling Back in "Live" Examples

For live games, we now present four tables, one for each different edge, the lowest being a 1% edge. We look at combinations of win ceilings and table maximums (\$500, \$1000, \$2000, \$2500, \$3000, \$5000). The table minimum is \$100. These represent realistic table limits found in American casinos. The chart shows the value of the game and the optimal bet depending also on the number of hands remaining. Bets are shown for a player who has \$10000, and for players whose initial \$10000 has been reduced to \$5000 or \$2000. In all cases, the win ceiling is computed relative to the initial starting bank of \$10000.

The value of the game increases with the edge, with a higher table limit, a higher win ceiling, and more hands remaining, as we would expect. Also, for a given wealth level (chip stack), we will

bet more if there are fewer hands remaining.

These examples are meant to be used as a guideline for blackjack and other games. The actual game used to generate the tables was our favorite game, Virtual Coin Toss. Winning bets are paid 3:2. This coin can land on its edge, creating a push.¹ The win probability and push probability are selected to match the expectation and variance desired (four charts, with player edges of 1%, 3%, 5%, and 7%, all with variance 1.44 on a 1-unit bet). We define a coin with our desired expectation and variance, but remember that the coin game has no doubling, splitting, or insurance, so additional funds are never required to supplement the initial bet.

If there is interest, we may examine betting strategies for the game of blackjack specifically, and also examine changes in *playing* strategy based on the CTR constraint. For now, we will let the numbers speak. In the forthcoming Part II of this article, we will see how the strategies and values change if a player starts with \$15000 or \$20000. The conclusion will be that players should save chips!

¹To avoid propagating rounding errors in the Bellman algorithm, described in *Rebated Losses Revisited*, we wanted the payoff to be an integer or half-integer. To then match the desired expectation and variance, we had to introduce the push probability. The formulas for the virtual coin are presented in Appendix E of *Beyond Counting 2005*.

Optimal Betting Facing CTR Constraint 1% Edge					
Table	Hands to Go				
Max	10	25	50	100	200
Win Ceiling=1000					
500	32:443/500/500	60:325/500/500	94:237/500/396	140:169/400/260	200:117/268/174
1000	40:499/1000/904	71:343/806/532	104:234/540/358	148:160/373/246	205:100/260/170
2000	45:489/1250/836	75:334/750/500	107:229/525/344	150:157/368/242	206:100/258/168
2500	46:490/1272/856	76:316/738/496	107:229/526/344	150:157/367/242	206:100/258/168
3000	46:488/1280/824	76:334/740/496	108:229/524/344	150:157/367/242	206:100/258/168
5000	46:497/1288/832	76:313/760/504	108:229/524/344	150:157/367/242	206:100/258/168
Win Ceiling=2500					
500	46:500/500/500	99:500/500/500	166:475/500/408	260:328/458/278	382:219/300/185
1000	70:1000/1000/908	131:664/924/564	195:442/606/380	281:304/415/258	396:211/289/178
2000	86:1020/1436/892	142:610/836/532	202:427/584/366	286:298/408/254	399:209/286/176
2500	88:1056/1382/872	143:609/834/524	203:424/580/364	286:298/407/254	399:209/286/176
3000	89:1002/1400/876	143:610/824/528	204:424/582/364	286:297/407/254	399:209/286/176
5000	90:970/1436/856	144:619/836/520	204:427/574/360	287:298/406/252	400:209/286/176
Win Ceiling=5000					
500	49:500/500/500	121:500/500/500	224:500/500/412	383:500/500/298	588:350/348/200
1000	92:1000/1000/920	192:1000/1000/604	298:702/702/404	435:475/474/278	621:329/328/190
2000	130:1638/1672/968	219:954/956/564	314:662/660/390	446:463/461/270	629:325/323/188
2500	135:1605/1608/928	222:945/948/560	316:657/658/388	448:461/460/270	630:325/323/188
3000	138:1512/1564/928	223:930/940/560	317:656/656/388	448:461/459/270	630:324/323/188
5000	141:1597/1560/960	225:936/940/560	319:654/656/388	449:461/459/268	631:324/322/188
Win Ceiling=10000					
500	50:500/500/500	124:500/500/500	248:500/500/416	473:500/500/314	816:500/432/224
1000	99:1000/1000/920	237:1000/1000/632	409:1000/870/460	627:717/569/310	915:487/387/210
2000	175:2000/2000/1024	314:1438/1148/628	459:978/780/436	658:680/541/296	936:476/378/206
2500	190:2436/1928/1012	322:1402/1120/616	464:967/772/424	662:676/538/294	939:475/377/204
3000	198:2320/1856/1016	326:1380/1104/608	466:962/768/420	663:674/537/294	940:474/376/204
5000	206:2320/1888/1064	331:1364/1084/600	470:950/764/416	666:672/535/292	942:473/376/204
Win Ceiling=20000					
500	50:500/500/500	124:500/500/500	249:500/500/416	489:500/500/316	917:500/492/236
1000	100:1000/1000/920	245:1000/1000/636	457:1000/996/476	788:1000/710/348	1228:697/475/240
2000	190:2000/2000/1036	394:2000/1436/704	610:1402/956/484	898:947/650/332	1299:657/450/230
2500	222:2500/2264/1068	421:2048/1388/704	627:1358/934/476	910:935/642/330	1307:652/447/228
3000	245:3000/2276/1108	435:1968/1348/696	636:1338/922/472	916:929/638/328	1311:650/446/228
5000	278:3186/2200/1104	453:1888/1296/672	648:1316/904/464	924:920/634/326	1318:647/444/226
Win Ceiling=50000					
500	50:500/500/500	124:500/500/500	249:500/500/416	489:500/500/316	922:500/496/236
1000	100:1000/1000/920	245:1000/1000/636	460:1000/1000/480	820:1000/760/356	1404:988/574/266
2000	190:2000/2000/1036	409:2000/1528/720	693:1984/1156/540	1127:1422/842/392	1745:952/578/274
2500	224:2500/2308/1080	461:2500/1600/748	759:2026/1184/552	1193:1376/832/392	1794:927/566/270
3000	252:3000/2324/1128	502:2828/1648/768	805:2006/1192/560	1229:1338/816/388	1819:915/560/268
5000	325:4536/2560/1208	590:2772/1680/796	878:1864/1140/548	1281:1286/788/380	1856:897/550/264
No Win Ceiling					
500	50:500/500/500	124:500/500/500	249:500/500/416	489:500/500/316	922:500/496/236
1000	100:1000/1000/920	245:1000/1000/636	460:1000/1000/480	820:1000/760/356	1405:996/576/266
2000	190:2000/2000/1036	409:2000/1528/720	693:1996/1156/544	1142:1522/870/396	1855:1155/650/294
2500	224:2500/2308/1080	461:2500/1600/748	764:2096/1204/556	1240:1594/902/408	1993:1204/673/302
3000	252:3000/2324/1128	503:2860/1652/768	820:2184/1244/568	1318:1654/930/424	2104:1243/692/308
5000	326:4640/2564/1208	612:3204/1808/824	970:2416/1352/612	1527:1810/1004/448	2398:1326/734/324
Table shows profit followed by optimal bets at chip stacks of 10000, 5000, and 2000. Win ceiling is strict. All figures represent dollars. Payoff is 3:2 with probability 0.386693. Push occurs with probability 0.0432667.					

Optimal Betting Facing CTR Constraint 3% Edge					
Table	Hands to Go				
Max	10	25	50	100	200
Win Ceiling=1000					
500	94:443/500/500	174:325/500/500	265:237/500/396	381:171/401/262	518:120/269/174
1000	117:499/1000/908	205:343/808/536	292:234/542/360	400:160/374/246	529:100/261/169
2000	133:489/1250/836	215:334/754/504	299:229/526/346	404:157/369/242	532:100/259/168
2500	134:490/1268/856	217:325/740/496	300:229/528/344	405:157/369/242	532:100/259/168
3000	135:488/1280/824	217:334/746/496	300:229/524/344	405:157/369/242	532:100/259/168
5000	135:497/1288/836	218:313/760/504	301:229/526/346	405:159/369/242	532:100/259/168
Win Ceiling=2500					
500	138:500/500/500	293:500/500/500	480:472/500/412	732:328/460/280	1035:220/301/184
1000	208:1000/1000/924	383:664/928/568	560:442/608/380	785:306/416/260	1068:213/289/178
2000	253:1024/1444/892	412:610/840/532	579:427/586/368	797:300/409/254	1075:211/287/176
2500	259:1056/1382/872	415:609/836/528	581:427/582/364	799:300/408/254	1076:210/287/176
3000	262:1002/1408/876	417:610/828/528	582:424/584/364	799:299/408/254	1077:210/286/176
5000	265:970/1436/856	419:619/836/520	583:427/576/364	800:300/407/254	1077:210/287/176
Win Ceiling=5000					
500	149:500/500/500	362:500/500/500	663:500/500/416	1110:500/500/302	1664:351/349/200
1000	276:1000/1000/928	569:1000/1000/608	872:705/704/408	1255:477/476/278	1750:331/328/190
2000	388:1647/1672/976	646:957/960/568	919:663/662/392	1286:464/463/272	1770:326/324/188
2500	402:1605/1608/928	654:948/950/560	924:659/660/390	1289:463/462/270	1772:326/324/186
3000	410:1512/1564/928	658:936/944/560	927:657/658/388	1291:462/460/270	1773:326/323/186
5000	418:1597/1560/960	663:936/944/564	930:656/658/388	1293:462/460/270	1775:326/323/186
Win Ceiling=10000					
500	150:500/500/500	374:500/500/500	744:500/500/420	1414:500/500/318	2417:500/436/226
1000	299:1000/1000/928	711:1000/1000/640	1224:1000/876/460	1869:720/571/310	2704:489/388/210
2000	525:2000/2000/1028	942:1446/1152/632	1373:982/784/436	1960:682/542/296	2764:478/379/204
2500	572:2440/1928/1024	965:1406/1124/616	1388:970/774/428	1971:678/540/296	2771:476/378/204
3000	593:2320/1856/1024	977:1390/1108/612	1396:964/770/420	1976:676/538/294	2775:476/377/204
5000	619:2320/1888/1064	993:1364/1088/600	1407:952/768/416	1984:674/536/294	2780:475/376/204
Win Ceiling=20000					
500	150:500/500/500	374:500/500/500	748:500/500/420	1476:500/500/320	2810:500/500/240
1000	300:1000/1000/928	737:1000/1000/644	1389:1000/1000/484	2422:1000/718/352	3807:700/477/240
2000	572:2000/2000/1036	1198:2000/1444/712	1868:1408/960/488	2770:951/652/334	4031:659/451/230
2500	671:2500/2304/1088	1283:2056/1392/708	1921:1364/936/476	2806:938/644/330	4057:654/448/228
3000	741:3000/2280/1112	1325:1980/1352/696	1949:1344/924/472	2825:932/640/328	4070:652/447/227
5000	843:3186/2208/1120	1382:1896/1304/676	1988:1320/908/464	2853:924/635/326	4090:649/445/226
Win Ceiling=50000					
500	150:500/500/500	374:500/500/500	748:500/500/420	1476:500/500/320	2834:500/500/242
1000	300:1000/1000/928	737:1000/1000/644	1398:1000/1000/484	2547:1000/776/364	4513:1000/588/268
2000	573:2000/2000/1036	1251:2000/1544/728	2158:2000/1168/544	3603:1438/850/396	5751:957/581/274
2500	678:2500/2324/1096	1418:2500/1616/752	2381:2052/1200/556	3831:1384/836/396	5924:931/568/270
3000	765:3000/2352/1124	1550:2868/1664/772	2534:2020/1200/560	3956:1344/820/388	6016:918/561/268
5000	995:4616/2564/1232	1835:2784/1688/800	2776:1874/1144/548	4134:1291/792/380	6148:900/550/262
No Win Ceiling					
500	150:500/500/500	374:500/500/500	748:500/500/420	1476:500/500/320	2834:500/500/242
1000	300:1000/1000/928	737:1000/1000/644	1398:1000/1000/484	2547:1000/776/364	4520:1000/591/270
2000	573:2000/2000/1036	1251:2000/1544/728	2160:2000/1172/544	3669:1558/884/402	6245:1187/663/296
2500	678:2500/2324/1096	1419:2500/1616/752	2399:2132/1220/560	4023:1626/918/416	6803:1235/686/304
3000	765:3000/2352/1124	1553:2904/1672/776	2590:2218/1260/572	4310:1686/944/426	7261:1273/704/312
5000	998:4652/2576/1232	1913:3240/1824/828	3113:2444/1364/616	5105:1836/1016/452	8525:1356/746/328
Table shows profit followed by optimal bets at chip stacks of 10000, 5000, and 2000. Win ceiling is strict. All figures represent dollars. Payoff is 3:2 with probability 0.39224. Push occurs with probability 0.0494.					

Optimal Betting Facing CTR Constraint 5% Edge					
Table	Hands to Go				
Max	10	25	50	100	200
Win Ceiling=1000					
500	154:443/500/500	280:319/500/500	414:237/500/400	574:171/402/262	742:117/270/174
1000	191:490/1000/912	325:343/812/536	452:234/544/360	598:160/375/246	755:117/262/168
2000	215:489/1250/836	341:334/758/504	462:229/528/346	604:160/370/242	758:117/260/167
2500	217:490/1268/856	343:334/744/496	463:229/529/346	605:160/370/242	758:117/260/166
3000	218:497/1280/824	344:334/746/500	464:229/526/344	605:160/370/242	758:117/259/166
5000	219:497/1288/836	345:313/762/508	464:229/528/346	605:160/370/242	758:117/259/166
Win Ceiling=2500					
500	230:500/500/500	480:500/500/500	769:469/500/412	1136:328/462/280	1542:222/302/184
1000	342:1000/1000/928	618:664/932/572	887:445/610/382	1211:306/418/260	1583:214/290/177
2000	413:1024/1448/892	664:610/842/532	915:430/587/368	1228:301/410/254	1593:212/288/176
2500	423:1056/1382/872	669:610/840/528	919:427/584/366	1230:301/410/254	1594:212/287/174
3000	428:1002/1408/876	671:610/830/528	920:427/584/364	1231:300/409/254	1594:212/287/174
5000	432:970/1436/856	674:619/840/524	922:430/578/364	1232:300/408/254	1595:212/287/174
Win Ceiling=5000					
500	249:500/500/500	601:500/500/500	1087:500/500/424	1778:500/500/304	2581:352/350/200
1000	458:1000/1000/932	933:1000/1000/616	1411:707/706/408	1994:478/477/278	2698:332/329/190
2000	640:1647/1672/976	1056:960/964/568	1485:666/664/392	2039:466/464/272	2725:328/325/186
2500	663:1605/1608/948	1068:951/952/560	1493:661/662/390	2044:465/463/270	2728:327/324/186
3000	676:1512/1564/936	1075:939/946/560	1497:660/660/388	2047:464/462/270	2730:327/324/186
5000	689:1598/1560/960	1084:936/944/564	1502:659/658/388	2050:463/462/270	2732:327/324/186
Win Ceiling=10000					
500	250:500/500/500	624:500/500/500	1239:500/500/424	2339:500/500/324	3927:500/440/227
1000	499:1000/1000/932	1182:1000/1000/644	2027:1000/882/464	3069:722/573/312	4363:491/389/210
2000	875:2000/2000/1028	1565:1450/1156/632	2271:985/786/436	3215:684/544/296	4454:479/379/204
2500	953:2440/1932/1032	1603:1410/1124/620	2295:973/776/428	3231:680/541/296	4464:478/378/204
3000	988:2320/1856/1024	1622:1394/1112/612	2308:967/772/420	3240:678/540/294	4470:477/378/203
5000	1032:2332/1888/1052	1649:1364/1088/600	2327:956/770/416	3252:676/537/294	4478:476/377/202
Win Ceiling=20000					
500	250:500/500/500	624:500/500/500	1248:500/500/424	2471:500/500/328	4743:500/500/246
1000	500:1000/1000/932	1232:1000/1000/648	2339:1000/1000/488	4107:1000/726/354	6448:703/478/240
2000	957:2000/2000/1040	2020:2000/1452/716	3165:1412/964/488	4703:954/654/334	6822:660/452/228
2500	1126:2500/2316/1096	2167:2064/1400/708	3257:1368/940/480	4765:941/646/330	6865:656/449/227
3000	1245:3000/2280/1112	2239:1984/1356/700	3304:1348/928/472	4798:935/642/328	6887:654/448/226
5000	1419:3192/2212/1120	2336:1900/1312/676	3371:1322/910/464	4844:927/636/326	6920:651/445/226
Win Ceiling=50000					
500	250:500/500/500	624:500/500/500	1248:500/500/424	2472:500/500/328	4809:500/500/246
1000	500:1000/1000/932	1232:1000/1000/648	2357:1000/1000/488	4373:1000/794/368	7975:1000/604/274
2000	959:2000/2000/1040	2120:2000/1560/736	3723:2000/1184/552	6358:1454/858/396	10363:961/583/276
2500	1139:2500/2328/1104	2418:2500/1636/764	4133:2076/1212/560	6785:1392/840/396	10690:934/570/270
3000	1288:3000/2368/1124	2653:2904/1680/780	4414:2036/1208/564	7019:1349/822/392	10864:920/562/266
5000	1690:4640/2604/1248	3166:2796/1696/800	4857:1880/1148/552	7350:1295/794/380	11113:902/552/262
No Win Ceiling					
500	250:500/500/500	624:500/500/500	1248:500/500/424	2472:500/500/328	4809:500/500/246
1000	500:1000/1000/932	1232:1000/1000/648	2357:1000/1000/488	4373:1000/794/368	7998:1000/608/274
2000	959:2000/2000/1040	2120:2000/1560/736	3728:2000/1188/552	6511:1596/900/406	11555:1223/678/302
2500	1139:2500/2328/1104	2419:2500/1636/764	4171:2168/1236/568	7212:1664/934/420	12765:1269/700/308
3000	1288:3000/2368/1124	2659:2948/1688/780	4532:2256/1280/576	7791:1720/958/432	13777:1306/718/316
5000	1694:4664/2608/1248	3315:3276/1840/844	5536:2480/1380/620	9434:1868/1028/456	16680:1389/760/332
Table shows profit followed by optimal bets at chip stacks of 10000, 5000, and 2000. Win ceiling is strict. All figures represent dollars. Payoff is 3:2 with probability 0.398. Push occurs with probability 0.055.					

Optimal Betting Facing CTR Constraint 7% Edge					
Table	Hands to Go				
Max	10	25	50	100	200
Win Ceiling=1000					
500	213:443/500/500	377:319/500/500	542:237/500/402	724:171/403/264	892:117/270/174
1000	260:490/1000/920	433:343/816/540	587:235/546/362	750:166/376/246	903:117/262/167
2000	291:489/1256/836	453:334/760/504	599:229/530/348	756:160/371/244	906:117/260/166
2500	295:490/1268/856	455:334/746/496	600:229/530/346	757:160/371/242	906:117/260/166
3000	297:497/1280/824	456:334/748/500	601:229/527/344	757:160/371/242	906:117/260/166
5000	297:497/1288/836	458:316/764/508	601:229/530/346	757:160/371/242	907:117/260/166
Win Ceiling=2500					
500	320:500/500/500	658:500/500/500	1032:463/500/416	1476:328/464/282	1917:223/303/184
1000	471:988/1000/932	836:664/936/572	1177:448/612/382	1561:307/419/260	1957:215/291/176
2000	567:1024/1448/896	895:618/842/536	1212:430/588/370	1580:303/411/254	1967:213/288/174
2500	579:1056/1382/880	902:618/842/528	1216:427/586/368	1583:301/411/254	1968:213/288/174
3000	586:1002/1408/896	905:610/830/528	1218:427/586/366	1584:301/410/254	1968:213/288/174
5000	592:970/1436/856	909:619/842/528	1220:430/578/368	1585:301/409/254	1969:213/288/174
Win Ceiling=5000					
500	349:500/500/500	837:500/500/500	1493:500/500/428	2379:500/500/308	3325:352/351/200
1000	638:1000/1000/936	1282:1000/1000/616	1912:708/708/412	2642:480/478/278	3453:333/330/188
2000	887:1683/1676/984	1446:963/964/568	2007:669/666/392	2697:468/465/272	3482:329/325/186
2500	918:1605/1608/940	1463:954/956/564	2017:663/662/390	2703:466/464/270	3485:328/325/186
3000	935:1512/1564/936	1472:942/948/560	2023:663/662/390	2707:465/463/270	3487:328/325/186
5000	953:1598/1560/960	1483:936/944/564	2029:662/660/388	2711:464/463/268	3490:328/324/186
Win Ceiling=10000					
500	350:500/500/500	874:500/500/500	1732:500/500/432	3238:500/500/330	5293:500/443/228
1000	698:1000/1000/936	1650:1000/1000/652	2811:1000/888/464	4200:723/574/312	5822:492/389/208
2000	1222:2000/2000/1032	2179:1456/1160/636	3141:988/790/440	4389:686/545/296	5931:480/380/204
2500	1331:2472/1944/1032	2232:1416/1128/624	3175:976/780/428	4411:682/542/296	5944:479/379/202
3000	1381:2338/1864/1028	2258:1398/1112/616	3192:970/774/424	4422:680/541/294	5951:478/378/202
5000	1443:2332/1888/1052	2296:1366/1092/604	3217:959/772/416	4438:678/538/292	5960:477/378/202
Win Ceiling=20000					
500	350:500/500/500	874:500/500/500	1748:500/500/432	3470:500/500/334	6677:500/500/250
1000	700:1000/1000/936	1728:1000/1000/656	3301:1000/1000/492	5810:1000/732/356	9025:705/480/240
2000	1344:2000/2000/1048	2857:2000/1464/720	4486:1416/968/488	6647:956/656/334	9523:662/452/228
2500	1585:2500/2328/1112	3068:2072/1404/712	4616:1372/942/480	6734:944/647/332	9578:658/450/226
3000	1756:3000/2300/1124	3171:1992/1364/700	4684:1350/930/476	6779:937/642/328	9608:656/448/226
5000	2005:3192/2208/1120	3309:1904/1316/676	4779:1324/912/468	6844:929/637/326	9651:653/446/224
Win Ceiling=50000					
500	350:500/500/500	874:500/500/500	1748:500/500/432	3473:500/500/334	6820:500/500/252
1000	700:1000/1000/936	1729:1000/1000/656	3334:1000/1000/496	6281:1000/816/376	11715:1000/621/278
2000	1347:2000/2000/1048	3015:2000/1576/740	5380:2000/1204/556	9360:1470/868/400	15419:965/586/276
2500	1606:2500/2336/1108	3458:2500/1652/768	6007:2098/1224/564	10016:1400/846/398	15912:937/571/270
3000	1821:3000/2384/1128	3810:2944/1696/784	6436:2048/1220/568	10374:1354/826/392	16174:923/563/266
5000	2407:4652/2600/1264	4577:2804/1700/804	7108:1886/1152/552	10877:1299/796/380	16547:904/552/261
No Win Ceiling					
500	350:500/500/500	874:500/500/500	1748:500/500/432	3473:500/500/334	6820:500/500/252
1000	700:1000/1000/936	1729:1000/1000/656	3334:1000/1000/496	6281:1000/816/376	11773:1000/628/278
2000	1347:2000/2000/1048	3015:2000/1576/740	5391:2000/1208/556	9653:1636/918/412	17755:1264/695/306
2500	1606:2500/2336/1108	3459:2500/1656/768	6077:2208/1252/572	10802:1706/948/424	19885:1308/716/314
3000	1821:3000/2384/1128	3820:3000/1704/784	6642:2298/1296/584	11764:1756/974/436	21702:1343/733/320
5000	2414:4692/2640/1264	4820:3316/1860/848	8246:2512/1396/632	14567:1902/1042/460	27101:1426/774/336
Table shows profit followed by optimal bets at chip stacks of 10000, 5000, and 2000. Win ceiling is strict. All figures represent dollars. Payoff is 3:2 with probability 0.403973. Push occurs with probability 0.0600667.					